PROGRESS REPORT No. 2

Theory of differential equations and their relationship to dynamical systems theory.

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BROWN UNIVERSITY

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I. INTRODUCTION

The chief areas of research at the present time are in the general classification of: (1) functional differential equations, (2) stability theory, (3) control theory, (4) stochastic differential equations, (5) network theory, and (6) finite state machines. Research accomplishments are reported on in Section III. For much of the work papers or reports have appeared or will appear and for a list of these see Section V.

This report summarizes the research of all members even though not all of it was supported by your office. Those members of the staff who have not received partial support from this contract are indicated by an asterisk in the list of staff members, Section II.

Scientific activities are reported on in Section IV and lectures given by visitors to the Center are listed in Section VI.

II. STAFF MEMBERS 1965-66

*Jose Arraut, Research Associate

*J. Billotti, Research Assistant

N. Chafee, Research Assistant

*M. A. Feldstein, Assistant Professor

J. J. Florentin, Associate Professor

*Robert Grafton, Research Assistant

Jack K. Hale, Professor

*James Heller, Postdoctoral Fellow[1]

Henry Hermes, Assistant Professor

*E. F. Infante Research Associate

H. J. Kushner, Associate Professor

J. P. LaSalle, Professor

Solomon Lefschetz, Visiting Professor

*Marshall Leitman, Research Associate

*Jorge Lewowicz, Resistant Assistant

Kenneth R. Meyer, Assistant Professor

*Z. Opial, Visiting Assistant Professor[2]

*A. E. Pearson, Assistant Professor

M. M. Peixoto, Associate Professor

Carlos Perello, Research Associate

*P. R. Sethna, Visiting Professor[3]

Leonard Weiss, Assistant Professor

F. Wesley Wilson, Research Associate

W. M. Wonham, Associate Professor

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1 Canadian National Research Council Post Ph.D. Fellowship

2 On leave from Krakow Instytut Matematyczny, U.J.

3 On leave from the University of Minnesota

*Did not receive support from this Grant.

III. RESEARCH RESULTS

A. Research Completed

1. Functional differential equations

a. Perello has filled some small gaps that remained in his doctoral dissertation and some improvements in the presentation have been made. A paper is being prepared for publication.

2. Stability theory

a. The exploitation of an invariance principle by LaSalle now makes clear the essential character of a Liapunov function and unifies the theory for those dynamical systems whose limit sets possess a suitable invariance property. For any arbitrary set G in the state space, the proper definition for a Liapunov function V on G is that V be C^1 on G and $\dot{V}$ not change sign. The principal theorem then states that V identifies all the possible positive and negative limit sets of solutions in G; i.e., a Liapunov function determines the asymptotic properties of all the solutions which remain in G. One then obtains all the usual Liapunov type theorems on stability and instability from this one theorem, and it has also already yielded a number of new theorems.

The following improvement of LaSalle's original theorem of this type has been obtained:

Theorem. Let G be a positively invariant compact set and let V be a Liapunov function (in the above sense) on G. Let E be the set in G where $\dot{V}$ vanishes and let M be the largest invariant set in E. If V is constant on the boundary of M, then M is asymptotically stable and G is in the region of asymptotic stability of M.

If, in the above theorem, M is a single point then the theorem says that this point is asymptotically stable. This completely dispenses with Liapunov's condition that V be positive definite. Even though a "good" Liapunov function should be positive definite and $\dot{V}$ should be negative definite, this result dispenses with both conditions and allows one to obtain results on asymptotic stability with "poor" Liapunov functions. This is important for applications because often "good" Liapunov functions are difficult to find. A complete report on these results is being prepared.

b. Infante has completed his study of the application of the second method of Liapunov to analyze the stability of surge-tanks in hydroelectric installations. The stability criteria he obtains are much less restrictive than previously available, and this study points out that the second method of Liapunov can be applied with considerable success to the determination of certain design criteria for complicated physical systems.

c. A fundamental set of theorems on finite time stability have been proved and published by Infante and Weiss.

d. A study of linear nonautonomous homogeneous n^{th} order equations by Infante has yielded a simple technique for the determination of sufficient conditions for the stability of the null solution. This technique is quite easy to apply and gives results comparable or better than the ones available in the literature.

3. Optimal nonlinear filtering

a. Consider the following filtering problem. Let the signal be the solution process of an Itô stochastic differential equation, and let the observed quantity be a nonlinear function of the signal added to 'white' Gaussian noise. For this nonlinear problem, dynamical equations governing the conditional moments of the process (the moments conditioned upon the observations) have been derived in the past by Kushner in a formal manner. Under checkable conditions on the signal process and on the form of the observation, the results have been substantiated in a rigorous form by Kushner. The results are believed to be the first proof of the optimality of the continuous time 'Kalman-Bucy' filter, and, in their nonlinear form are expected to be of use for nonlinear filtering problems. Some simulation and numerical studies of the filter have been made, and are continuing. A paper on this has been submitted by Kushner to the Journal of Differential Equations.

4. Stochastic control

a. Wonham has completed a paper entitled "A Lyapunov method for the estimation of statistical averages". This paper has been accepted for publication in the Journal of Differential Equations.

b. Using some results of Dynkin, Kushner has obtained sufficient conditions for the optimality of a stochastic control (to control an Itô process to a given target set and minimize the average value of an integral of a positive

'rate of loss' function, from t_o to the random arrival time). His results are a type of extension of the deterministic results using the Caratheodory Theory of the Calculus of Variations. His results also give conditions under which a partial differential equation commonly derived via dynamic programming actually yields the optimal loss function and the optimal control. They are, in this sense, a justification of the dynamic programming result. A minimum 'average' time problem is worked. Unlike results of this type which may be investigated via the theory of elliptic equations, ours do not depend on ellipticity and are valid for deterministic systems as well. A paper by Kushner is to appear in the SIAM J. on Control.

c. A report has been written by Kushner giving a stochastic Liapunov function method for obtaining estimates of the probability

$$P_x\{\sup_{T\geq t\geq 0} V(x_t) \geq \lambda\} .$$

$V(s)$ is a suitable function. Such probabilities are first passage time probabilities and ones of basic importance in many engineering applications. A number of Theorems are stated and proved and examples worked in CDS TR 65-7, October, 1965.

5. Control theory

a. Hermes has given a sufficient condition for optimality for linear time optimal controls and a paper on this subject has been submitted to Journal of Mathematical Analysis and Applications.

b. Hermes has completed a study of discontinuous vector fields and has prepared a CDS Technical Report on his research which will appear in the near future. Briefly, this report studies the effect of perturbations $\varepsilon(t)$ which enter a differential equation with discontinuous vector field X as $\dot{x}(t) = X(x(t) + \varepsilon(t))$. If X is a C^1 field, such a perturbation is equivalent to an additive perturbation; this is far from the case when X is allowed discontinuities, as for instance often occurs in feedback control problems. Since the perturbation enters in such a way as to alter a measurement of the state $x(t)$ of a system at time t, the terminology stability with respect to measurements has been introduced and intuitively states: the vector field X is stable with respect to measurement if a solution φ of $\dot{x} = X(x)$, $x(0) = x^0$, exists on $[0,T]$ for arbitrary initial data x^0 and whenever $\varepsilon(t)$ is such that a solution ψ of $\dot{x} = X(x + \varepsilon(t))$, $x(0) = x^0$ exists, $\psi(t)$ will remain arbitrarily close to $\varphi(t)$ if $|\varepsilon(t)|$ is sufficiently small. The relations between vector fields which are stable with respect to measurement, Filippov generalized solutions, and implications to control theory are studied.

6. Stability of generalized flows

Seibert has completed a study of the stability of autonomous dynamical systems characterized as continuous flows in a suitable state space (a locally compact metric space). In terms of the notion of "prolongation" of orbits beyond their limit sets a necessary and sufficient condition is given that a compact set M be stable in the sense of Liapunov. A characterization is also given in terms of wandering points of the region of attraction of a compact positively invariant set.

B. Continuing research

1. Functional differential equations

a. A paper by Hale is near completion on the asymptotic behavior of the solutions of systems of linear functional differential equations. By employing some of the geometric properties of the solutions, the work of Bellman and Cook (Memoires of the AMS, No. 35, 1959)) is generalized and greatly simplified.

b. The work begun by Hale on boundary value problems for functional differential equations has run up against some difficult problems and all that has been accomplished to date is a clarification of the hypotheses made by Halanay in his approach to this problem.

c. Perello is continuing his research on the application of the generalization of the Cesari-Hale method for the determination of periodic solutions of differential equations with time lag when the nonlinearities are small. He is now working on several examples. He is also considering the possibility of being able to exploit symmetries in the way Hale did for ordinary differential equations.

d. Meyer has returned to a consideration of the problem of the existence of solutions to a degenerate system of linear differential difference equations. Equations of this type occur in singular perturbation problems for differential difference equations. He has shown that solutions exist in general provided the initial date is sufficiently smooth, namely, C^{∞}. Furthermore, if the spectrum of the linear operator associated with the equation is restricted he has shown that the solutions exist even if the initial data is only C^k where the integer k is a measure of the degeneracy of the equation. This last result establishes a close relationship between the phenomenon of "loss of derivatives" and the existence of advanced chains of zeros to the characteristic equation.

e. Chafee has considered a dynamical system in E^n of the form

$$\begin{aligned} \dot{y} &= A(\epsilon)y + Y(x, t, \epsilon), \\ \dot{z} &= C(\epsilon)z + Z(x, t, \epsilon), \end{aligned} \tag{1}$$

where $x \in E^n$ and $x = (y, z)$ with $y \in E^2$, $z \in E^{n-2}$. Also, ϵ is a parameter varying in an interval $[0, \epsilon_o]$. It is assumed that (1) $C(\epsilon)$ is a $(n-2) \times (n-2)$ matrix whose eigenvalues have real parts less than $-\alpha$ for some $\alpha > 0$, (2) $A(\epsilon)$ has the form

$$A(\epsilon) = \begin{pmatrix} a(\epsilon) & -b(\epsilon) \\ b(\epsilon) & d(\epsilon) \end{pmatrix}$$

where $b(\epsilon) > 0$ and $d(\epsilon) \to 0+$ as $\epsilon \to 0+$ and $\alpha/2 > a(\epsilon)$, and (3) Y and S are second order in (y, z) and have period ω in t and are such that the origin $x=0$ is asymptotically stable when $\epsilon = 0$.

Under these general conditions as well as some more restrictive hypothesis, Chafee has shown that for ϵ sufficiently small the origin bifurcates into an asymptotically stable manifold which relative to the appropriate coordinate system is homeomorphic to a two dimensional cylinder about the t-axis. Furthermore, this cylinder is periodic in the sense that any two cross sections separated by a distance ω are exactly alike.

In recent weeks he has also treated the bifurcation of an asymptotically stable closed orbit into an asymptotically stable torus and a slight generalization of this problem. The solution of this problem has the merit of closely paralleling the procedure used in the above problem.

Finally, it appears likely that using the techniques developed for the bifurcation problem discussed in connection with (1) he will be able to develop a bifurcation theory for a certain type of differential difference equation whose form resembles (1).

2. Stability theory

a. Further work on the theory of finite time stability developed by Weiss and Infante is being pursued. A student of Weiss is presently developing some results in this area for systems containing a forcing function and is work-

ing on the development of converse theorems for finite time stability. Some new results on finite time stability in product spaces are available and their possible applications are being investigated.

b. A study of the stability of n^{th} order homogeneous linear differential equations with time varying coefficients has been completed by Infante. At the present time the results obtained are being extended to nonlinear systems with moderate success.

c. Fr. Billotti has begun a study of the asymptotic behavior of solutions of periodic and almost periodic nonlinear systems of differential equations when these systems are asymptotically or almost autonomous. This study includes a consideration of invariant and quasi-invariant limit sets. No new results have been obtained. Since the results are not well-known and would appear to provide new methods of studying the stability properties of such systems, it would seem worthwhile to attempt to apply the theory to some significant examples in order to illustrate that the theory is applicable and to show how it can be applied.

3. Control theory

a. Weiss is continuing to explore further aspects of the theory of controllability and observability. Modifications and extensions of the existing theory are being made, with resulting further insight into the problem of relating various representations of linear systems.

4. Network theory

a. Lefschetz has continued to occupy himself intensely with grasping the recent work of Brayton and Moser on nonlinear networks. It has struck him that in this paper, and in many writings on graph theory, the work done for years by topologists has been totally ignored. He has tried to remedy the situation by initiating the writing of a monograph with Weiss and Infante and has incidentally been able to find a number of interesting results; one of these will appear as a Note in the Proceedings of the National Academy for December under the title "On planar graphs and related topics". Briefly, by utilizing the passage to two-dimensional topology he has been able to derive well-known, necessary and sufficient conditions for imbedding graphs in a plane and also the extension of this result to the imbedding of graphs in higher surfaces (these last results are entirely new).

b. As an illustration of the usefulness of applications of algebraic topology to the theory of finite graphs and networks, Weiss has given a new derivation of the well-known Tellegen theorem in which the concepts of exterior algebra, chains, co-chains, and derivation operator play fundamental roles.

5. Qualitative theory

a. Peixoto has obtained a considerable simplification in the proof of a theorem of Kupka and Smale to the effect that on a compact manifold M^n any system can be approximated by a system with generic singularities and closed orbits such that the corresponding stable and unstable manifolds do intersect transversely. Still under study is the possibility of extending this important theorem to non-compact manifolds; this can be done for sure if $n = 2$. In any case a manuscript containing these investigations will be submitted to the Journal of Differential Equations.

b. Concerning an investigation by Peixoto mentioned in a previous Progress Report about the closing Lemma in class $r \geq 1$, dimension 2, a flaw in the proof was found so that the manuscript containing that result is not yet ready. In any case, out of this investigation Peixoto has obtained some new results. For instance he can now prove that on any compact M^2 any nontrivial minimal set of a field of class C is "essentially" obtained from the classical example of Denjoy on the torus; i.e., by putting on it "artificial" holes and singularities and making identifications.

c. During this period Wilson has continued to investigate possible applications of differential topology to the qualitative behavior of ordinary differential equations. The most recent outgrowth of these investigations is a characterization of the level lines of a Liapunov function up to homotopy type. The characterization up to homeomorphism class seems to be equivalent to the Poincare conjecture.

6. Discrete dynamical systems (theory of machines)

a. Florentin has continued his effort to find an effective way of mathematically describing discrete state, discrete time, dynamic systems which are exemplified by a computer. Three approaches are being considered

(i) To try to find parametric specifications of a subset of the partial recursive functions, thus defining a class of systems from a

computer program viewpoint;

(ii) Setting up state models of machines, with the states being elements in some algebraic structure such as a group. The input/output relations for the machine are now defined by the algebraic structure.

(iii) Attempting to apply the ideas of computable algebras introduced by Rabin. An algebraic structure is chosen; this will have a number of functions associated with it; for instance, a group may have a homomorphism onto a quotient group. These functions can often be shown to be recursive, and thus the algebraic structure is again a means of defining a class of recursive functions.

The core of the problem is to define an interesting class of machines. Empirically machines have been classified by the way they store numbers and there are finite store machines, infinite pushdown store machines, infinite random store machines and some others. Mathematical methods exist for finite machines, but practical interest centers on infinite machines; here no mathematical methods are known, in particular no way of making the mathematical formalism correspond to a storage type is known. What is needed is some notion of what mathematical properties are implied by empirical characteristics of machines.

b. A paper by Florentin and B. C. Chartres entitled "A Universal Syntax Analyser" has been submitted to the Communications of the ACM. This paper describes an empirical algorithm which acts as the characteristic function of the set of all words in any given context-free language. Because of the practical usefulness of context-free languages many algorithms have been constructed; however, all published algorithms failed in some circumstances. Since it can be shown theoretically that a perfect algorithm must exist there is pedagogical value in demonstrating it.

C. <u>New Research</u>

1. Functional differential equations

a. In a paper "Linear functional differential equations with constant coefficients", Contributions to Differential Equations, II (1963), 291-317, Hale by studying in detail the eigenspaces of linear functional differential equations and making use of the adjoint equations was able to introduce new coordinates in terms of which it was possible to develop a perturbation theory similar to that for ordinary differential equations. He is now attempting to extend the methods used there to functional differential equations of neutral

type; that is, those in which the future state of the system is determined not only by its present and past history but also the rate of change of the past history.

b. Along these same lines Perello has begun to consider how these results of Hale (referenced above) can be generalized so as to apply to functional differential equations with infinite time hereditary effects.

Perello has also started work on the generalization of Cesari's method for the determination of periodic solutions of nonlinear ordinary differential equations to functional differential equations. The object is to prove that under some restrictive hypothesis periodic functional differential equations have periodic solutions if some finite set of determining equations can be solved. This plus an implicit function theorem would be the basis for the justification of the Ritz-Galerkin method for differential equations with time lag.

c. Feldstein is studying retarded ordinary differential equations (RODE) of the form $y'(x) = f(x, y(x), y(\alpha(x)))$ where $a \le \alpha(x) \le x \le b$ and $y(a)$ is given. This restriction upon $\alpha(x)$ has the primary purpose of making the RODE one where the initial set reduces to a point ($\alpha(a) = a$), thus requiring the least amount of initial information from which to construct the solution. In addition it focuses attention upon the inherent problems that occur when $\alpha(x)=x$.

He, in particular, is looking at the problem of the construction of approximations to solutions $y(x)$; that is, the numerical analysis aspects of the RODE problem. The difficulty here arises from the following:

Suppose one defines a mesh $x_n = a + nh$, $n=0,1,\ldots,n$ where $Nh = b-a$ and $h > 0$. Suppose further that one tries to apply the alogorithm of Euler's method to define a sequence $\{y_n\}$ as

$$y_o = y(a)$$

$$y_{n+1} = y_n + hf_n$$

(One expects that y_n would "approximate" $y(x_n)$, the true solution evaluated on the mesh.) How does one choose f_n ? For ordinary differential equations one takes $f_n = f(x_n, y_n)$. For RODE one has a third argument to approximate; that is, one needs to approximate $y(\alpha(x_n))$ in a manner similar to the approximation that y_n makes to $y(x_n)$. Here $\alpha(x_n)$ will rarely be a mesh point. Furthermore, after one chooses some approximation for the third argument, one must determine how this affects the error. In addition the algorithm chosen must be readily

handled by digital computers. For instance, an algorithm that requires infinitely many evaluations of an indefinite integral (such as the method of Picard) simply would not be practical.

This numerical analysis problem, though important, has been virtually unsolved in any systematic manner. There does exist the idea of El'sgolt's to generate a variable mesh $x_{n+1} = x_n + h_n$ so choosen that $\alpha(x_n)$ will be an "old" mesh point. A few simple examples readily show that this method is not fruitful. For convergence of the El'sgolt's algorithm it is necessary that $\max h_n$ tend to zero. If one considers $y'(x) = y(\frac{x}{2})$, $y(0) = 1$, then this condition is not satisfied. Furthermore, it is clear that one needs to overspecify the initial condition, for otherwise f_1 could never be evaluated. There are other difficulties too. If $y'(x) = y(x^2)$, $y(0) = 1$, $x \in [0,1]$, then El'sgolt's algorithm can never approximate $y(1)$ because the sequence $\{x_n\}$ is strictly increasing and bounded above by 1. Furthermore, if one had several retardations, as in $y'(x) = y(\frac{x}{2}) + y(x^2)$, then one could not in general choose a mesh so that $\alpha_i(x_n)$ is an old mesh point for each i.

Thus Feldstein's efforts are directed towards algorithms that are easy and practical to apply, that will apply systematically, that include the complications of $\alpha(x) = x$ and of $\alpha(x)$ increasing indefinitely with x, that include several retardations, that will generalize for systems of RODE and that will certainly work for the simple examples above.

He has considered certain first order one step algorithms and has shown that under suitable hypotheses they converge uniformly as finite intervals to the solution $y(x)$. The discretization error has been shown to be bounded (the bound, while a function of f, is independent of α) as $|y_n - y(x_n)| \leq Mh$ where M depends upon the smoothness of f and the interval $[a, b]$.

A parameter study of the algorithms has been made which leads to a more basic simplified algorithm. It too is convergent and its error is similarly bounded.

Feldstein has also explored the question of the error more deeply to understand how it depends upon f and α. For one of the algorithms a theorem was proved stating conditions for the existence of an asymptotic error expansion. Further, a specific representation is given.

Numerical experiments have confirmed the convergence theorems as well as the order of convergence. The asymptotic error expansion has also been confirmed by experiments. For one of the algorithms, while numerical results have indicated the likelihood of the existence of an asymptotic error expansion, he

has not been able to prove it (yet!).

The simplified algorithm is the most intriguing and the most difficult. At best he can conjecture about an error expansion. It would have to be in fractional powers of h, the exponent being $1 + ks$, where $k=0,1,2,\ldots$ and $0 < s \leq 1$ with s such that polynominals of degree $\frac{1}{s}$ (or the smallest integer $\geq \frac{1}{s}$) vanish on $[a,b]$ faster than does α' (the derivative of α). Thus $\alpha(x) = x^{\lambda+1}$ would be expected to have an expansion of $h \cdot h^{\lambda}$, $h \cdot h^{2\lambda}$ etc.

The key to the conjecture is some other work of his on a perturbation theorem for RODE and certain asymptotic order estimates. The validity of the conjecture would seem to depend on certain number-theoretic properties of the fractional part of $\frac{\alpha(x)-a}{h}$.

At present Feldstein's research for the RODE problem is directed toward the following:

(1) Asymptotic error expansions--filling the holes mentioned above.

(2) Alternate algorithm.

(3) Higher order one-step methods.

(4) Multi-step methods, both implicit and explicit.

(5) Modification of the RODE as posed.

Regarding (1) the progress is slow, the work is hard and there may or may not be a reasonable solution. Nevertheless, this is most fascinating and would be theoretically rewarding. Item (2) progresses as ideas occur. Both (3) and (4) seem to have good promise. The road is not without trouble, but the results seem to be forthcoming and would be highly useful. Number (5) is explored as necessity arises and several results have been obtained.

d. Meyer has recently become interested in the problem of asymptotic integration for functional differential equations. He hopes to carry on some of the work started by Jack Hale.

e. Leitman, who joined the Center this fall, has started an investigation of the spectrum of the infinitesimal generator operator associated with linear functional differential equations. In particular he is attempting to relate the spectrum associated with an "infinite lag" equation with that of "finite lag". These results should illuminate some of the qualitative differences, if any, between solutions of the two types of equation.

f. In 1962, G. Stephen Jones proved the existence of periodic solutions of the equation: $f'(x) = -\alpha f(x-1)\{1 + f(x)\}$. Grafton is investigating the existence of these periodic solutions more deeply with the hope of discovering methods for determining periodic solutions of more general functional differential equations.

2. **Stability theory**

a. During the last month Infante has become interested in some problems of the stability of mechanical structures which are dynamically loaded. It appears that many of the techniques developed for the application of the second method of Liapunov are applicable to problems of this type. At the present time he is studying the literature available on this subject and has successfully attacked one simple problem by a Liapunov technique to obtain results which are superior to those previously known ones.

b. Since the invariance principle (see Section A, 2, above) appears to be the key to a unified theory of stability, a general formulation of the principle to encompass partial differential equations would seem to be worthwhile. In order to do this in such a manner that the application to particular classes of dynamical systems (such as those defined by partial differential equations) is apparent it would seem to be necessary to formulate this theory for a general type of differential equation rather than for a generalized flow. For example, Zubov's formulation which was intended to be applicable to partial differential equations covers only a few quite special types of these equations. From Hale's extension of the invariance principle to functional differential equations one would conjecture that the invariance principle depends upon the dynamical system having the property that it "smooths" the initial data (the output is smoother than the input in the sense of continuity and differentiability). LaSalle is undertaking a study of this problem and this has a bearing on the application of Liapunov's method to the dynamic stability of structures mentioned above.

3. Stochastic stability

a. Wonham has begun research on finite time stochastic stability in connection with a problem which arose in the course of consulting activities at NASA. This study involves the application of approximation theory to the first passage time problem for a dynamical system perturbed by noise.

4. Stochastic optimal control

a. Heller, who joined the Center in October, is continuing research along the lines of his Ph.D. dissertation. He is now in the process of preparing an article (and possibly two) based upon the results of his dissertation on stochastic optimal control processes. One article deals with the derivation of an algorithm for determining the optimum controller and mean cost rate for a continuous time (discrete time as well), continuous space Markovian optimization problem based upon the quasilinearization technique of dynamic programming and certain assumptions (viz. ergodicity and positivity). Under contemplation is a second article concerning the optimal control of a linear system with random parameters and a quadratic, integral performance index. The nonlinear optimum filtering equations in the case of noisy observations are derived for the above system. The dual control problem is discussed.

5. Qualitative theory

a. Arraut has just recently joined our staff and has begun to study for differentiable actions of R^n on R^{n+1} the way in which a periodic orbit can be approached by orbits of the same action.

b. Lewowicz, who joined the Center this fall, is investigating the asymptotic behavior of dynamical systems inside a compact region D which contains a compact invariant set and which has the property that each solution starting in D has an antecedent outside D . He will investigate first the analytic case. He is also beginning a study of generic vector fields.

6. Partial differential operators

Hermes has started an investigation of the possible application of the Choquet representation theorem and its extension as given by E. Bishop and K. DeLeeuw, to characterize properly posed problems for certain classes of partial differential operators.

IV. SCIENTIFIC ACTIVITIES

During the month of May Hermes gave a lecture on control theory at the Esso Research Center, Florham Park, New Jersey and LaSalle gave a lecture on stability theory at Massachusetts Institute of Technology.

Pearson was the co-author with Philip E. Sarachik of Columbia University of a paper that was recently awarded a second prize at the Sixth Joint Automatic Control Conference. The title of the paper was "On the Formulation of Adaptive Optimal Control Problems".

Weiss presented a paper on finite time stability, co-authored by Infante and himself, at the national SIAM meeting in New York in June. In August he gave a talk on finite time stability at the 23rd Space Flight and Guidance Seminar at the NASA installation in Huntsville, Alabama.

Hermes and Meyer were visiting lecturers during the summer at the University of British Columbia, Vancouver, Canada. Hale gave a series of lectures on functional differential equations during the summer at the Centro de Investigacion y de Estudios Avanzados del IPN in Mexico. LaSalle gave an address in June on the present status and future prospects of the theory of stability and control at the Air Force Office of Scientific Research's Scientific Seminar held at Cloudcroft, New Mexico.

LaSalle also gave a series of lectures from September 6-11 at the NATO Summer Institute at the University of Padua, Padua, Italy. After this trip he visited as a guest of the Soviet Union various institutes within the Soviet Union and during the latter part of September he attended the Third All-Union Conference on Control and Cybernetics held on the Black Sea. He presented a paper at that meeting on "An Invariance Principle in the Theory of Stability". He also visited institutes and universities in Warsaw and Krakow and gave lectures at the Mathematics Institutes in both Warsaw and Krakow.

In October Lefschetz lectured on control theory in the lecture series on differential equations sponsored by the Air Force Office of Scientific Research at Georgetown University, Washington, D.C. LaSalle gave an invited address on "Stability by Liapunov's Direct Method" at the International Conference on the Dynamical Stability of Structures held in October at Northwestern University, Evanston, Illinois. This conference was sponsored jointly by the Air Force Office of Scientific Research and The Technological Institute at Northwestern.

On October 20 Lefschetz presented a paper on graph theory at the Allerton Conference on Circuit and Systems Theory at the University of Illinois and on the same day he gave a talk to the Department of Mathematics at the University on control theory.

Wonham has been a consultant for the NASA Electronics Research Laboratory in Cambridge and has assisted them in contract monitoring and proposal reviewing. Wonham and Florentin are participating in an interesting experimental new course on control systems which is being offered this year to advance students in the University's Medical Sciences Program. Wonham is also preparing a detailed review of stochastic control theory which will appear as a chapter in a book being edited by A. T. Bharucha-Reid to be published by Academic Press.

V. PUBLICATIONS

Hale, Jack K.

"Sufficient conditions for stability and instability of autonomous functional differential equations", J. Diff. Eq., 1 (1965) 452-482

"Linear asymptotically autonomous functional differential equations", submitted to J. Math. Anal. Appl.

Hermes, Henry

"The equivalence and approximation of optimal control problems", J. Diff. Eq., 1 (1965) 409-426.

"Sufficiency in linear time optimal control", submitted to J. Math. Anal. Appl.

Infante, E. F.

"The simulation of the differential equations of a simple surge-tank", to appear in L'Energia Elettrica (Milan, Italy)

"On the stability of systems defined over a finite time interval", Proc. N.A.S., 54 (1965) 44-48.

"On the stability of the oscillations of a simple surge-tank" (with L.G. Clark), to appear Trans. ASME Ser. E. J. Appl. Mech.

"Stability criteria for N-th order, homogeneous linear differential equations", to appear in Proc. of Int'l Symposium on Differential Equations and Dynamical Systems. December, 1965. (Academic Press)

Kushner, Harold J.

"On the construction of stochastic Liapunov functions", to appear IEEE, Trans., G-AC

"A note on the maximum sample excursions of stochastic approximation processes", to appear Ann. Math. Statist.

"Finite time stochastic stability and the analysis of tracking systems", to appear IRE Trans. Auto.Control.

"Dynamical equations for optimal nonlinear filtering", submitted to J. Diff. Eq.

LaSalle, J. P.

"Stability and optimal control", Proc. of IBM Scientific Computing Symposium on Control Theory and Applications, Yorktown Heights, N.Y., Oct. 19-21, 1965.

"The stability of nonlinear dynamical systems", AFOSR Anniversary Book, Science in the Sixties (June, 1965 at Cloudcroft, N.M. - Univ. of N.M. Press)

Lefschetz, S.

"On planar graphs and related topics", to appear as a Note in Proc. NAS December, 1965.

Meyer, Kenneth R.

"On the existence of solutions to linear differential difference equations", to appear in Proc. of Int'l Symposium on Differential Equations and Dynamical Systems, Dec., 1965 (Academic Press).

(with K. L. Cooke) "The condition of regular degeneration for singularly perturbed systems of linear differential difference equations", to appear in J. Math. Anal. Appl.

"On the existence of Lyapunov functions for the problem of Lur'e", to appear in SIAM J. on Control (Ser. A).

(with W. Baran) "The effects of delayed neutrons on the stability of a nuclear power reactor", to appear in Nuclear Science and Engineering.

Weiss, Leonard

"Contributions to linear system theory" (with R.E.Kalman), Int'l J. Engrg. Sci., 3 (1965) 141-171

"The concepts of differential controllability and differential observability", J. Math. Anal. Appl. 10 (1965) 442-448

"Dual dynamical systems and their representation by system functions", Int'l J. Control, 1 (1965) 475-485

"On system functions with the property of separability", Int'l J. Control, 1 (1965) 487-496

"On the stability of systems defined over a finite time interval", Proc., NAS, 54 (1965) 44-48 (This paper with E. F. Infante)

Wilson, F. Wesley

"On the minimal sets of nonsingular vector fields", submitted to J. Math. Anal. Appl.

"The structure of the level lines of a Liapunov function", submitted to J. Diff. Eq.

Wonham, W. M.

"Lyapunov criteria for weak stochastic stability", to appear J. Diff. Eq.

"A Lyapunov method for the estimation of statistical averages", to appear J. Diff. Eq.

VI. LECTURES BY VISITORS

Arenstorf, Richard F. NASA - George C. Marshall Space Flight Center	May 19-20	Perturbation theory and celestial mechanics
Athans, Michael MIT, Elec. Engrg. Dept.	April 13	On the uniqueness of the extremal controls for a class of minimum fuel problems
Bhatia, Nam P. Western Reserve University	April 2	Weak-attractors
Coppel, W. A. Australia - and U.S.Army Math. Research Center, Univ. of Wisconsin	April 30	Systems of homogeneous equations
Cuenod, Michel Visiting Professor Univ. of Florida	May 5	Power control systems
Grenander, Ulf Univ. of Stockholm and Brown University	June 2	Adaptive control in the presence of noise
Pallu de la Barriere, R. Univ. of Caen, Laboratory of Automation Theory	April 7	Pontryagin's maximum principle in the case of constraints involving simultaneously control and state variables
Pugh, Charles Univ. of California, Berkeley	April 9	The closing lemma and structural stability
Sandberg, I. W. Bell Telephone Laboratories	June 17	Some results on the theory of physical systems governed by nonlinear integral equations
Zames, George Mass. Institute of Technology	May 13	Functional analysis: approach to the input-output stability of nonlinear systems